Divisor topology and GV invariants of CICYs

Federico Carta

Durham University

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Based on...

1. Divisor topologies of CICY 3-folds and their applications to phenomenology
   - F.C, A. Mininno, P. Shukla. 2022
2. Gopakumar-Vafa hierarchies in winding inflation and uplifts
   - F.C, A. Mininno, N. Righi, A. Westphal. 2021

Motivation

- One of the main goals of String Phenomenology is to make concrete string models which:
  1. Reproduce the esablished low-energy physics (SM, ΛCDM...)
  2. Extend the esablished low-energy physics (DM, SUSY, Inflation...)
- Type II string theory on $\mathbb{R}^{1,3} \times X_6$, with X_6 compact CY 3-fold.
- 4d EFT is $\mathcal{N} = 2$ SUGRA. More realistic $\mathcal{N} = 1$ EFTs arise from modding out by an orientifold action $\Omega\mathcal{R}(-1)^{F_L}$
- Focus on IIB, with orientifold action allowing for $O3/O7$ planes. Typical setup for the flux landscape (GKP '01. KKLT '03)
- Properties of the CY orientifold fix properties of the low energy 4d EFT.

Complex structure moduli stabilization

Prepotential has a polynomial part, and instanton corrections

$$\mathcal{F}(U) = \mathcal{F}_{\text{poly}}(U^i) + \mathcal{F}_{\text{inst}}(U^i) \tag{1}$$

$$\mathcal{F}_{\text{inst}} = \frac{1}{(2\pi i)^3} \sum_{d_i} A_{d_i} e^{2\pi i d_i U^i} \tag{2}$$

- GVs n^g_β (naively) count the number of holomorphic maps from worldsheet to a curve in the class $\beta = q^i \beta_i \in H^2(\tilde{X}, \mathbb{Z})$.

$$\begin{aligned} \mathcal{F}_{\text{inst}}(U^i) &= \frac{1}{(2\pi i)^3} \sum_{\beta \in H_2^-(\tilde{X}_3, \mathbb{Z}) \setminus \{0\}} n_\beta \, \text{Li}_3(q^\beta) \\ \text{Li}_3(x) &= \sum_{m=1}^{\infty} \frac{x^m}{m^3}, \quad q^\beta = e^{2\pi i d_i U^i} \end{aligned} \tag{3}$$

Complete intersection CYs

- Defined as the zero locus $\tilde{X}$ of a set of homogeneous polynomials $p_i[x], (i = 1, \dots K)$ in an ambient space $\mathcal{A} = \mathbb{P}^{n_1} \times \dots \mathbb{P}^{n_s}$
- Multidegrees of $p_i[x]$ encoded in the configuration matrix.

x^i	$\mathbb{P}^2$	0	2	0	1
y^i	$\mathbb{P}^2$	2	1	0	0
w^i	$\mathbb{P}^3$	1	1	1	1

$$\sum_{i=1}^{s} n_i - K = 3$$

$$\begin{aligned} p_1[x] &= a_{ijk} y^i y^j w^k, \\ p_2[x] &= b_{ijkl} x^i x^j y^k w^l, \\ p_3[x] &= c_i w^i, \\ p_4[x] &= d_{ij} x^i w^j. \end{aligned} \tag{4}$$

- List of at most 7890 (possibly) distinct CY constructed in this way. (Candelas, Dale, Lutken, Schimmrigk '87) (Green, Hubsch, Lutken '89) (Anderson, Gao, Gray, Lee '17)

Computing the genus 0 GV by Mirror symmetry

(Hosono, Klemm, Theisen, Yau, '94)

- $\Pi(z) = \left(w_0(z), \frac{\partial}{\partial \rho_i} w_0(z,\rho) \mid_{\rho=0}, \dots \right)^t$
- $w_0(z) = \sum_{n_1 \geq 0} \dots \sum_{n_{h^{2,1}} \geq 0} c(n) \prod_{i=1}^{h^{2,1}} z_i^{n_i}$.
  Generic solution of the PF equation for the first entry of the period vector, in terms of data in the configuration matrix.
- $w_i(z) =$
  $\sum_{n_1 \geq 1} \dots \sum_{n_{h^{2,1}} \geq 0} \frac{1}{2\pi i} \frac{\partial}{\partial \rho^i} c(n+\rho) \mid_{\rho=0} \prod_{i=1}^{h^{2,1}} z_i^{n_i} + w_0(z) \frac{\ln z_i}{2\pi i}$
- $t^i = \frac{w_i(z)}{w_0(z)}$ mirror map. Relation between Kahler moduli at large radius in the A-model side, with complex structure moduli at LCS in the B-model side.
- Now, invert the mirror map in order to get $z(t)$. Hardest step.

Computing the genus 0 GV by Mirror symmetry

(Hosono, Klemm, Theisen, Yau, 94)

- Having $z(t)$, compute the quantum corrected triple intersection number

$$k_{ijk} = \frac{\partial}{\partial t_i}\frac{\partial}{\partial t_j}\frac{\frac{1}{2}k^0_{kab}\frac{\partial}{\partial \rho_a}\frac{\partial}{\partial \rho_a}w_0(z,\rho)\mid_{\rho=0}}{w_0(z)}(t)$$

- Introduce $q_i = \exp(2\pi i t_i)$, and the general expression for the quantum corrected triple intersection number

$$k_{ijk} = k^0_{ijk} + \sum_{n_1\geq 1}\dots\sum_{n_{h^{2,1}}\geq 0} n_{d_1,\dots,d_{\bar{h}^{1,1}}} d_i d_j d_k \frac{\prod_{l=1}^{\bar{h}^{1,1}} q_l^{d_l}}{1-\prod_{l=1}^{\bar{h}^{1,1}} q_l^{d_l}}$$

- Match and extract the GV.

Instanton program, and the scan

- The above algorithm has been coded in a program called Instanton. (Klemm, Kreuzer)
- Made modification of their code, parallelized it, and let it run.
- Computing time $\approx$ 6 months, on two different clusters (DESY and Madrid IFT)
- List all the genus 0 GV, up to total degree 10, for all favourable CICY up to $h^{1,1} = 9$.
- We find directions in the Picard lattice in which the GV invariants grow at hierarchical rates, as well as "vanishing directions" (Demirtas, Kim, McAllister, Moritz '20) and "periodic directions".
- List at www.desy.de/∼westphal/GV_CICY_webpage/GVInvariants.html

Occupation sites for CICY 7858 ($h^{1,1} = 2$)

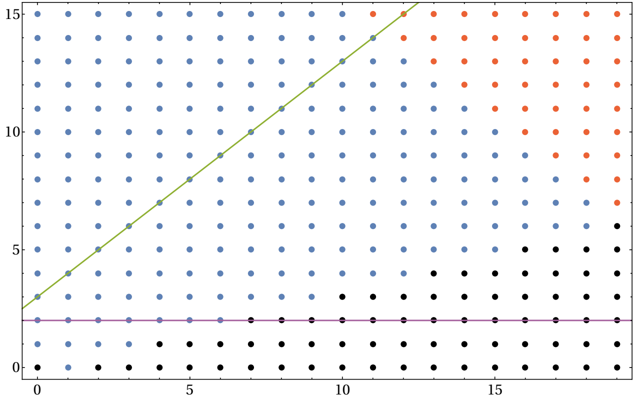

Figure: Blue=non-zero GV. Black = zero GV. Orange = not computed, believed to be non-zero GV. Green = non-vanishing direction. Purple = vanishing direction.

The need for rigid divisors

- Suppose we have fully stabilized the c.s. moduli. At lower energy if there are no non-perturbative effects, Kahler moduli have a flat potential (no scale). $W = W_0 := \langle W_{GVW} \rangle$
- Non-trivial potential for Kahler moduli can be generated by the presence of Euclidean E3s or D7-branes.

$$W(T) = W_0 + \sum_{\vec{n}} A_{\vec{n}} e^{-2\pi n_a T^a} \quad (5)$$

- However, not any holomorphic 4-cycle will do the job.
- Relevant to ask: does your compactification space admit such nice 4-cycles?

Rigidity, ampleness

Rigidity

- In order to generate non-perturbative contribution to the superpotential, E3 or D7 need to wrap a rigid divisor.
- Rigid = no normal bundle deformations. $\mathcal{O}(D)$ has a unique section. $h^{2,0}(D) = 0$.
- $\chi_h(D) = 1$ is a necessary condition for nontrivial W_{np}.

Ampleness

- If a divisor is ample (and rigid) a single E3 or D7 wrapped on them can stabilize all saxions of the T^a and a combination of the C_4 axions. (Bobkov et al. '21)

$$W_{np} = A \exp\left[-i \sum_{\alpha=1}^{h^{1,1}_+} a_\alpha T_\alpha\right] \tag{6}$$

More on ampleness

- (Nakai-Moishezon) A divisor D is ample if for every closed subvariety $Y \subset X$
$$D^{\dim(Y)} \cdot Y > 0 \quad (7)$$
- Another way to define it is a codimension one surface with ample canonical bundle. (i.e. opposite of del Pezzo surfaces). Also called *surface of general type*.
$$c_1^2 = \int_D c_1(D) \wedge c_1(D) \; > 0, \quad c_2 = \int_D c_2(D) \; > 0. \quad (8)$$
- Some properties
$$h^{p,q}(X) = h^{p,q}(D), \quad \forall\, p + q < 2 \quad (9)$$
$$\pi_i(X) = \pi_i(D), \quad \forall\, i < 2 \quad (10)$$

Divisor topologies

- We want to discuss what is the topology of divisors D of CICYs.
- Solve this problem for coordinate divisors (i.e. $x_i = 0$).
- $h^{ij}(D)$ can be computed via Koszul spectral sequence, and knowledge of $H^i(X, D)$.
- Can use Cohomcalg to do this computation. (Blumenhagen et al. '10, '11) However, starting from $h^{1,1}(X) = 5$, it gets very slow.

Idea!

$$\chi(D) = 2h^{0,0} - 4h^{1,0} + 2h^{2,0} + h^{1,1} = \int_X \left(\hat{D} \wedge \hat{D} \wedge \hat{D} + c_2(X) \wedge \hat{D} \right) \quad (11)$$

$$\chi_h(D) = h^{0,0} - h^{1,0} + h^{2,0} = \frac{1}{12} \int_X \left(2\, \hat{D} \wedge \hat{D} \wedge \hat{D} + c_2(X) \wedge \hat{D} \right) \quad (12)$$

Conjecture connected, simply connected.

- Conjecture that, for all favorable CICYs, $h^{0,0}(D) = 1$ and $h^{1,0}(D) = 0$. Connected and simply connected.
- Conjecture verified by explicit computation up to $h^{1,1}(X) = 6$. Have counterexamples for non-favourable CICYs.
- Now trivially solve for $h^{1,1}(D)$ and $h^{2,0}(D)$.

$$D \equiv \begin{array}{ccccc} & & 1 & & \\ & 0 & & 0 & \\ (\chi_h - 1) & & (\chi - 2\chi_h) & & (\chi_h - 1) \\ & 0 & & 0 & \\ & & 1 & & \end{array} \tag{13}$$

Results

Sr. #	Divisor topology $\{h^{0,0}, h^{1,0}, h^{2,0}, h^{11}\}$	frequency (57885 divisors)	frequency (7820 spaces)	$h^{1,1}$ (pCICY)	$\int_{CY} \hat{D}^3$
T1	$K3 \equiv \{1,0,1,20\}$	30901	7736	2-15	0
T2	$\{1,0,2,30\}$	22150	7436	2-15	0
T3	$\{1,0,3,38\}$	3372	2955	2-13	2
T4	$\{1,0,3,36\}$	91	91	3-13	4
T5	$\{1,0,4,46\}$	714	690	2-11	4
T6	$\{1,0,4,45\}$	283	277	1-11	5
T7	$\{1,0,4,44\}$	91	91	2-11	6
T8	$\{1,0,5,52\}$	198	198	1-9	8
T9	$\{1,0,5,51\}$	28	28	1-9	9
T10	$\{1,0,6,58\}$	42	42	1-7	12
T11	$\{1,0,7,64\}$	15	15	1-5	16

Table 2: Divisor topologies for favorable pCICYs and their frequencies of appearance.

Conclusions and future directions

Summarizing the results:

- Compute explicitly genus zero GV up to degree 10 for all favourable CICYs up to $h^{1,1} = 9$.
- Classify topologies of coordinate divisors in CICYs. None of them is rigid.

For the future:

- Does something funny happen for non-favourables? Schoen etc.
- Interesting to perform similar tasks in KS.
- Topologies of non-coordinate divisors.
- Global models on CICYs in generic toric ambient spaces and on gCICYs are essentially unexplored.